The Effects of Business Uncertainty on Investment Policies of Financial Intermediaries

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In this paper I tested the effects that the business environment uncertainty has on the investment decisions of companies within financial industry. More specifically I tried to determine the effects of either high or low volatility of business environment on investment policies of financial intermediaries, such as banks, pension funds and insurance companies. As the results of two demonstrative examples indicate increasing the volatility of future losses/payments (e.g. future losses for insurance company and payments into a pension fund) of the financial intermediary results in a more risky investment strategy even under a very risk averse optimization criterion. This could indicate that small companies, which have in general a higher coefficient of variation of payments/losses than bigger companies, should hold more risky asset.

Key words: economic organization, banking&insurance, optimal portfolio allocation

Vpliv negotovosti na naložbeno politiko finančnih posrednikov

V članku preučujem vpliv negotovosti v poslovnem okolju na naložbeno politiko finančnih posrednikov (zavarovalnice, pokojninski skladi itn.). Bolj natančno preučim kako volatilnost prihodnjih obveznosti vpliva na alokacijo med tvegane in ne-tvegane naložbe, pri čemer kot mero tveganja uporabim VaR (Value at Risk). S pomočjo numeričnih simulacij ugotovim, da se ob večji negotovosti prihodnjih obveznosti za finančne posrednike optimalna naložbena politika premakne v smer tveganih naložb. To lahko pomeni, da naj bi majhni posredniki, ki imajo višji koeficient variacije vplačil proti izplačilom (obveznostim), v portfeljih imeli višji delež tveganih naložb.

Ključne besede: gospodarska organizacija, bančništvo&zavarovalništvo, optimalna naložbena politika

1 Introduction

In this paper I focus on the effect that the business environment uncertainty has on the investment decisions of companies within financial industry. More specifically I try to determine the effects of either high or low volatility of business environment on investment policies of financial intermediaries, such as banks, pension funds and insurance companies. In order to quantify these effects I solve two general multi-period portfolio models from the fields of insurance and finance under both the assumptions of high and low volatility of business environment. By comparing the results of numerical calculations for the examples considered I try to deduce what are the effects of volatility of business environment on investment policies of financial companies[1].

First, I analyse the problem of a portfolio investor who has to meet a series of future random payments/losses X_i by investing an amount K under some investment strategy $\vec{\pi}$ defined by an allocation between different asset classes. In choosing the optimal investment strategy I look for an investment mix that, given a fixed default probability p, minimises the initial capital investment K.

[1] Some of the more prominent types of models from the area of multi period portfolio selection include Brennan et al. (1997, 1998, 1999, 2002), Campbell et al. (1996, 1999[a,b], 2000, 2001[a], 2001[b], 2003), Xia (2001), Xia et al. (2001), Merton (1971, 1973), Samuelson (1967, 1969), Rubinstein (1976[a,b], 1981) and Stiglitz (1970).

The second problem I analyse is a general multi-period portfolio problem of a financial agent (either a personal investor or a financial intermediary such as a bank or a mutual fund) that wants to maximise the amount of future wealth K by periodically investing some amounts X_i according to an investment strategy $\bar{\pi}$. In order to keep the analysis as general as possible, I assume that the invested amount is random and allow for a serial correlation between successive payments. In choosing the optimal investment strategy I look for an investment mix that maximises the amount of accumulated wealth[2], which is achieved with a sufficiently high probability p (e.g. 95%).

The asset dynamics are modelled within the well-known Black & Scholes setting, which assumes log-normally distributed asset prices. I assume that the investor has to choose the optimal investment strategy given a predictable consumption/saving pattern, whereby the optimality of investment strategies is as mentioned defined via VaR(p) or quantile risk measure.

The structure of this paper is as follows. In Section 2 I explain the basic setup of the model (Black & Scholes settings, constant mix portfolios). In Section 3 I introduce the saving for retirement problem. The reserving problem which is in some sense dual to terminal wealth problem is presented in Section 4. Section 5 gives the results of the simulation and numerical illustrations along with some comments. The final remarks are discussed in Section 6.

2 Model setup

In this section I give the main characteristics of the model (such as the quantile risk measure, dynamics of the market etc).

2.1 Quantile as a risk measure

In simple terms a risk measure gives a description of riskiness of a random variable by summarizing the information contained in the distribution function in one single real number. Amongst the risk measures the most widely used is the quantile risk measure or Value at Risk (VaR).

For a given random variable K the p-quantile risk measure (VaR) is defined by

$$Q_p\left[X\right]=\inf\left\{x\in R \mid F_X(x)\geq p\right\}, \qquad p\in(0,1), \tag{1}$$

where $F_X(x)=\Pr\left[X\leq x\right]$. A related risk measure is denoted by $Q_p^+[X]$ and is defined by

$$Q_p^+\left[X\right]=\sup\left\{x\in R \mid F_X(x)\leq p\right\}, \qquad p\in(0,1). \tag{2}$$

Observe that only values of p corresponding to a horizontal segment of F_X lead to different values of $Q_p[X]$ and $Q_p^+[X]$. Thus when F_X is strictly increasing, both risk measures will coincide for all values of p. In this case, one can also define the $(1-p)$-th quantiles by

$$Q_{1-p}\left[X\right]=\sup\left\{x\in R \mid \overline{F}_X(x)\geq p\right\}, \qquad p\in(0,1), \tag{3}$$

where $\overline{F}_X(x)=1-F_X(x)$. For more about the relationship between different risk measures see Dhaene et al. (2003).

2.2 Market dynamics

In describing the market dynamics I adopt the so called Black & Scholes framework (see Black et al., 1973)

2.2.1 The Black & Scholes setting

Consider a market of $n+1$ securities which are traded openly and can be bought or sold without incurring any cost. One of the assets is assumed to be risk free, the others are risky. The price of the risk-free asset evolves according to the following deterministic (ordinary) differential equation

$$\frac{\mathrm{d}P^{(0)}(t)}{P^{(0)}(t)}=r\mathrm{d}t, \tag{4}$$

where r stands for the drift or return of the risky asset. Thus the price of the risk-free asset grows exponentially and can be given explicitly by

$$P^{(0)}(t)=P^{(0)}\exp(rt), \tag{5}$$

with $P^{(0)}$ denoting the amount that was invested at time 0.

Other assets are assumed to be risky in the sense that their price is not deterministic and evolves according to a following stochastic differential equation. The price process $P^i(t)$ evolves according to a geometric Brownian motion stochastic process, represented by the following stochastic differential equation:

$$\frac{dP^i(t)}{P^i(t)}=\mu_i dt+\sum_{j=1}^{d}\overline{\sigma}_{ij}dW^j(t), \tag{6}$$

with $u_i>r$ the drift of the i-th risky asset and $\left(W^1(s), W^2(s), \ldots, W^d(s)\right)$ a d-dimensional standard Brownian motion process. Here it is assumed that the $W^1(s)$ are mutually independent standard Brownian motions.

The diffusion matrix $\overline{\Sigma}$ is defined by

$$\overline{\Sigma}=\begin{pmatrix} \overline{\sigma}_{11} & \overline{\sigma}_{12} & \cdots & \overline{\sigma}_{1d} \\ \overline{\sigma}_{21} & \overline{\sigma}_{22} & \cdots & \overline{\sigma}_{2d} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{\sigma}_{m1} & \overline{\sigma}_{m2} & \cdots & \overline{\sigma}_{md} \end{pmatrix} \tag{7}$$

whereas the matrix Σ (referred to also as the variance-covariance matrix) is defined as

[2] Problems within a similar framework are analysed in Milevsky et al. (1997), Milevsky and Robinson (2000) and Dhaene et al. (2004).

$$\Sigma = \bar{\Sigma} \times \bar{\Sigma}^T = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_m^2 \end{pmatrix}, \tag{8}$$

with coefficients σ_{ij} and σ_i^2 given by $\sigma_{ij} = \sum_{k=1}^{d} \bar{\sigma}_{ik} \bar{\sigma}_{jk}$ and $\sigma_i^2 = \sigma_{ii}$. Observe that $\sigma_{ij} = \sigma_{ii}$ hence the matrix is symmetric. Additionally we assume that Σ is positive definite. Thus for all non-zero vectors $\pi^T = (\pi_1, \pi_2, \cdots, \pi_m)$ we have that all σ_i are strictly positive and that Σ has a matrix inverse.

If one defines the process $B^i(s)$ by

$$B^i(s) = \frac{1}{\sigma_i} \sum_{j=1}^{d} \bar{\sigma}_{ij} W^j(s). \tag{9}$$

than equation (6) can be rewritten as:

$$\frac{dP^i(t)}{P^i(t)} = \mu_i dt + \sigma_i dB^i(t). \tag{7}$$

Observe, that in contrast to equation (6) where $W^i(\tau)$ are uncorrelated standard Brownian motions, the $B^i(\tau)$ are correlated standard Brownian motions, with

$$Cov\left(B^i(t), B^j(t+s)\right) = \frac{\sigma_{ij}}{\sigma_i \sigma_j} t, \qquad t, s \ge 0. \tag{8}$$

The solution to equation (7) is

$$P^{(i)}(t) = P^{(i)} \exp((\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i B^{(i)}(t)), \tag{9}$$

with P^i as before denoting the price of i-th risky asset at time 0.

From equation (9) one finds the price of the risky asset to be log-normally distributed with the first two moments given by

$$E[P^{(i)}(t)] = P^{(i)} \exp(\mu_i t), \tag{10}$$

$$Var[P^{(i)}(t)] = (P^{(i)})^2 \exp(2\mu_i t)(\exp(\sigma_i^2 t) - 1). \tag{11}$$

A more detailed representation of a multidimensional return process in a Black & Scholes setting can be found in e.g. Björk (1998) or Dhaene et al. (2004).

2.2.2 Constant mix investment strategies

In this Section I briefly recapitulate some of the most important results on the topic of constant mix investment strategies.

As before, consider a market of n risky and one risk-free security. Within this setting, any investment strategy can be characterised by an allocation vector $\vec{\pi}(t) = (\pi_0(t), \pi_1(t), \ldots, \pi_n(t))^T$, with $\vec{\pi}_i(t)$ denoting the percentage of the i-th risky asset held at time t and $\pi_0(t)$ the percentage of risk-free asset in the portfolio. Observe that the fraction placed in the risk-free asset is determined by the aggregate percentage of all risky assets in the portfolio

$$\vec{\pi}_0(t) = 1 - \sum_{i=1}^{n} \pi_i(t). \tag{12}$$

In the case of a constant mix investment strategy, the percentages (in terms of value) of different assets remain constant over time, so that the time component can be dropped

$$(\pi_0(t), \pi_1(t), \ldots, \pi_n(t))^T = (\pi_0, \pi_1, \ldots, \pi_n)^T. \tag{13}$$

Although the proportions of each asset type are independent of time, the portfolio nevertheless has to be continuously rebalanced in order to keep the percentages of each asset type constant. This strategy implies a "buy low and sell high" principle. Namely, if a price of an asset falls while the prices of all other assets remain constant, one should increase the quantity of that stock (which has fallen) and reduce the quantity of other securities to maintain a constant mix within one's portfolio.

Given a class of constant mix strategies $\vec{\pi}$ one can prove that the portfolio price process $P(t)$ evolves according to the following stochastic differential equation

$$\frac{dP(t)}{P(t)} = \sum_{i=1}^{m} \pi_i \frac{dP^i(t)}{P^i(t)} + \left(1 - \sum_{i=1}^{m} \pi_i\right) \frac{dP^0(t)}{P^0(t)} \tag{14}$$
$$= \left(\sum_{i=1}^{m} \pi_i (\mu_i - r) + r\right) dt + \sum_{i=1}^{m} \pi_i \sigma_i dB^i(t).$$

If we introduce a process $B(\tau)$ by

$$B(\tau) = \frac{1}{\sqrt{\vec{\pi}' \times \Sigma \times \vec{\pi}}} \sum_{i=1}^{m} \pi_i \sigma_i B^i(\tau). \tag{15}$$

It can be shown that $B(\tau)$ is a standard Brownian motion, so that we can rewrite equation (14):

$$\frac{dP(t)}{P(t)} = \mu(\vec{\pi}) dt + \sigma(\vec{\pi}) dB(t), \tag{16}$$

with $B(t)$ a standard Brownian motion and $\mu(\vec{\pi})$ and $\sigma^2(\vec{\pi})$ defined as

$$\mu(\vec{\pi}) = r + \vec{\pi}^T \cdot (\vec{\mu} - r1) \quad \text{and} \quad \sigma^2(\vec{\pi}) = \vec{\pi}^T \cdot \Sigma \cdot \vec{\pi}. \tag{17}$$

Here 1 denotes the m-dimensional vector of ones $(1, 1, \ldots, 1)^T$ and Σ stands for a variance-covariance matrix which is assumed to be positive definite. The solution of equation (16) is

$$P(t) = P \exp((\mu(\vec{\pi}) - \frac{1}{2}\sigma^2(\vec{\pi}))t + \sigma(\vec{\pi}) B(t)), \tag{18}$$

with expectation and variance given by

$$E[P(t)] = P \exp(\mu(\vec{\pi}) t), \tag{19}$$

$$Var[P(t)] = P^2 \exp(2\mu(\vec{\pi}) t)(\exp(\sigma^2(\vec{\pi}) t) - 1). \tag{20}$$

Throughout this Section I use the concept of yearly return, which gives the log-value of one money unit investment after a one-year period. In line with equation (18) a return in year can be written as

$$Y_k(\vec{\pi}) = \mu^{'}(\vec{\pi}) + \sigma(\vec{\pi})(B(k) - B(k-1)), \tag{21}$$

with $\mu^{'}(\vec{\pi})$ (where $\mu^{'}(\vec{\pi})$ is equal to $\mu(\vec{\pi}) - \frac{1}{2}\sigma^2(\vec{\pi})$) denoting the drift, $\sigma(\vec{\pi})$ standard deviation (on a yearly basis) of investment strategy $\vec{\pi}$ and $B(k)$ stands for a standardised Brownian motion. In more general terms, the value of a single unit investment over a period of k years expressed in terms of yearly returns can be written as

$$P(k) = P\exp(Y_1(\vec{\pi}) + Y_2(\vec{\pi}) + \cdots + Y_k(\vec{\pi})). \tag{22}$$

Observe that the yearly returns $Y_1(\vec{\pi})$ are independent and normally distributed; hence the return over a period of k years is also normally distributed.

3 Saving for a retirement problem

As mentioned, the first problem I address is the so called saving for retirement problem. More precisely, I consider a multi-period portfolio problem of a decision-maker who wants to maximise the *p*-quantile of terminal wealth distribution function. In order to achieve this goal he periodically invests random amounts $X_1, X_2, \ldots, X_n$ (where the considered amounts are on a yearly basis and measured in real terms, i.e. adjusted for inflation) over the length of the investment horizon. I assume that the investment amounts X_i are normally $N(\mu,\sigma^2)$ distributed. The logic behind such an assumption lies in the stochastic nature of the investor's environment. For example, private investors saving for their retirement are exposed to idiosyncratic shocks (such as the loss of a job or a sudden injury) which can result in a negative balance between that year's earnings and consumption. It thus makes sense also to model examples where one allows for the net investment to be negative with some small probability *p*.[3]

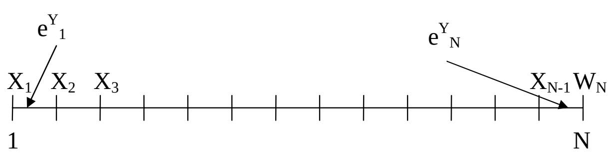

Figure 1: Saving for a retirement problem

The amounts X_i are invested according to some constant mix investment strategy $\vec{\pi}$ as described in the previous Section. The goal of the investor is to choose such a strategy, that the amount of the end wealth $W_n(\vec{\pi})$ guaranteed with sufficiently high probability p is maximized. As one can prove; this goal is obtained by maximizing the $1 - p$ quantile of end wealth.[4]

$$F^{-1}_{W_n(\vec{\pi})}(1-p) = \sup\left(x \in R; \Pr\left[W_n(\vec{\pi}) \geq x\right] = p\right) \tag{23}$$

Thus the optimisation criterion is set equal to:

$$\max_{\vec{\pi}}(F^{-1}_{W_n(\vec{\pi})}(1-p)). \tag{24}$$

4 The reserving problem

In the second problem I consider the case of an investor who has to meet a series of future random payments/losses X_i by investing an amount K under some investment strategy $\vec{\pi}$. The random payments are assumed to be lognormally distributed $LN(\mu,\sigma^2)$, so that the payments in each year are strictly positive. In choosing the optimal investment strategy I look for an investment mix $\vec{\pi}$ that, given a fixed default probability $1 - p$, minimises the initial capital investment K.[5]

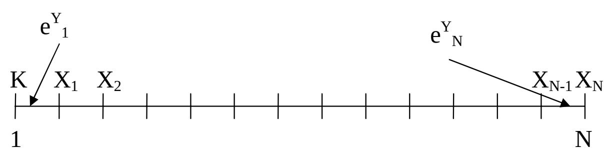

Figure 2: The reserving problem

In obtaining this goal the investor chooses such a strategy $\vec{\pi}$ that minimizes the p quantile of the present value of future payments, where the discounting is done with respect to investment returns $Y_n(\vec{\pi})$. More precisely, one can show that setting the initial capital K equal to

$$K = F^{-1}_{W_n(\vec{\pi})}(p) \tag{25}$$

fulfills the condition that the default probability is at most $1 - p$.

$$\Pr\left[K \cdot e^{Y_1(\vec{\pi})+..+Y_n(\vec{\pi})} - \sum_{i=1}^{n-1} X_i \cdot e^{Y_i(\vec{\pi})+..+Y_n(\vec{\pi})} - X_n \geq 0\right] = F^{-1}_{W_n(\vec{\pi})} \tag{26}$$

5 Numerical illustration

In this Section the results of numerical calculations are presented. In all the examples considered the same set of parameters describing market dynamics is selected; drift of the stock market index $\mu_m = 0.073$ and standard deviation $\sigma_m = 0.16$, for the drift of the risk-free account I took $r = 0.012$ whereas for the length of the investment horizon

[3] A similar problem is discussed in Ahčan et. al. (2005).

[4] The following expression is valid for the case of contionous and increasing cumulative distribution function $F_{W_n(\vec{\pi})}(x)$.

[5] A similar problem is discussed in Ahčan (2004, 2005).

$T = 40$ was chosen[6] (I perform ED 300 000 simulations for each of the 15 investment strategies considered).

5.1 Terminal wealth problem

In this subsection I discuss the results for the terminal wealth problem considered in Section 3 I present the results for the case of low (almost constant payments) and high volatility (very unstable environment) of yearly payments. In the first case I assume the payments X_i to be normally distributed with mean $\mu_X = 10$ and standard deviation $\sigma_X = 0.5$, whereas in the second example the standard deviation is set equal to $\sigma_X = 0.5$, while the mean stays the same. Additionally I assume successive payments to be serially correlated, with the correlation equal to 0.5 if the time lag between successive payments is 1 year and 0.2 if the lag is 2 years. All other correlations are assumed to be zero.

In Figure 3 I present the results for the first case. One can observe that the maximum is achieved for an investment strategy equal to $\pi_{opt} = (27.5\%, 72.5\%)^T$ (where the first value denotes a fraction invested in the risk-less asset and the second value denotes the fraction invested in the market portfolio) with the corresponding 95% guaranteed terminal wealth equal to $K = 578.5$.

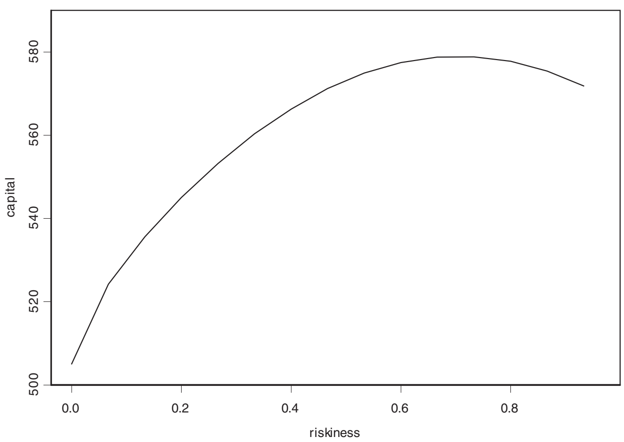

Figure 3: The 0.05-quantile of the terminal wealth cumulative distribution function with respect to the riskiness of an investment strategy (var=0.25)

Figure 4 presents the results of the second example (variance 25 units). In this case the optimal investment strategy is achieved in the case of $\pi_{opt} = (20.5\%, 79.5\%)^T$ with the corresponding terminal wealth equal to $K = 558.5$.

Having evaluated both of the numerical examples (where now the only difference is with regard to the volatility of payments) allows one to make some conclusions about the optimal investment policies under uncertainty. As one can see, increasing the volatility of payments to the investment fund decreases the 95% guaranteed accumulated wealth and increases the asset mix towards more risky assets. If it is an intuitive result that increasing the volatility of payments decreases the 95% guaranteed accumulated wealth, then the amount of decrease is certainly surprising; increasing the volatility of payments by a factor of 100 decreases the amount of wealth which is achieved with a probability of 95% by a mere 3.5%.

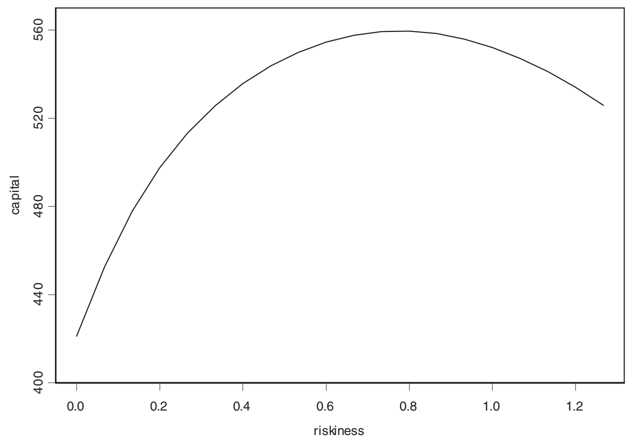

Figure 4: The 0.05-quantile of the terminal wealth cumulative distribution function with respect to the riskiness of an investment strategy (var=25)

Another striking result is that although the optimisation criterion is very risk-averse (95% of the outcomes should be in our favour) increasing the randomness in the investment environment (by increasing the volatility of payments) pushes the optimal investment mix towards risky assets. To explain this effect, one should examine how the volatility of the terminal wealth distribution changes as one increases the volatility of payments. Clearly, if the volatility of payments is relatively small most of the terminal wealth volatility comes from the stochastic nature of the financial environment (i.e discount factors). In this case, the overall volatility of accumulated wealth is mostly determined through the choice of investment strategy and it makes sense under a risk-averse objective function to pick a more risk-free investment strategy. On the other hand, when the volatility of payments is larger the investment strategy will not have a prevailing effect of reducing the volatility of accumulated wealth since both of the random vectors (payments, investment returns) will contribute to the overall volatility. Accordingly, it makes sense to choose a riskier investment strategy with a higher expected return since then below-average payments can still be expected to accumulate enough wealth through higher expected investment returns. Although some of the outcomes will be less favorable (when under-average payments are accompanied by poor investment returns) the result shows that the overall effect of going more risky will be predominantly positive (even under strict risk-averse criteria such as a 5% VaR).

[6] As shown in Ahčan (2005) the analysis can be restricted to a subset of two assets: a risk free asset and a market index.

5.2 The reserving problem

In this subsection I present the results for the reserving problem, with log-normally distributed losses. Two cases are considered: the case of low volatility of losses (the effect of pooling is significant and future losses can be regarded as almost constant), and the case of high volatility (in the case that the background factors affecting risk can not be effectively diversified or the company has not a large enough pool of risks, the volatility of future losses is significant).

In the first example the choice of parameters is $\mu_X = 2.3$ and $\sigma_X = 0.05$, and for the second example $\mu_X = 2.2$ and $\sigma_X = 0.46$. The choice of parameters in both of the scenarios is such that the first moments (mean) are equal to 10 in both cases while the variances differ significantly (approximately *var* = 25 for the second scenario vs. *var* = 0.25 for the first scenario).

In Figure 5 I present the results for the first scenario (small volatility of losses). The optimal portfolio weights calculated by means of Monte Carlo simulation are $\pi_{opt} = (64\%, 36\%)^T$ (where as before the first value denotes a fraction invested in the risk-less asset and the second value denotes the fraction invested in the market portfolio), which corresponds to an initial capital investment *K* = 303.5.

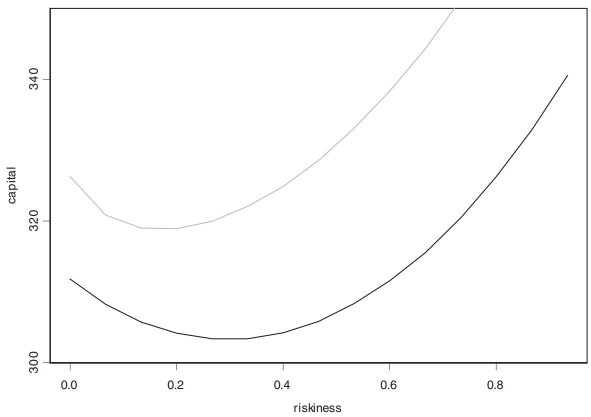

Figure 5: The 0.95-quantile of the cumulative distribution function with respect to the riskiness of an investment strategy (var = 0.25).

In Figure 6 the results of the second scenario are presented. The optimal portfolio weights are equal to $\pi_{opt} = (57.5\%, 42.5\%)^T$ and the corresponding initial investment is equal to *K* = 309.9.

Comparing both results, a similar conclusion can be drawn as before. Namely, in the case of more volatile future losses a higher provision (that guarantees at most 5% probability of default) needs to be established. Between the two cases a relatively small difference is observed; in the case of higher volatility of future losses provision is higher for a mere 2%. Again, as before a similar line of reasoning can be applied. Namely, if the volatility of losses is relatively small the amount of provision necessary to cover future losses will be mostly affected by the volatility of the discount factors. In this case, the overall volatility of present value of future losses is mostly determined through the choice of investment strategy and it makes sense to select a more risk-averse investment strategy. On the other hand, when the volatility of losses is larger the investment strategy will not have a prevailing effect of reducing the volatility of present value of future losses. In this case a riskier investment strategy has to be chosen to offset the effects of less favorable results in the loss portfolio.

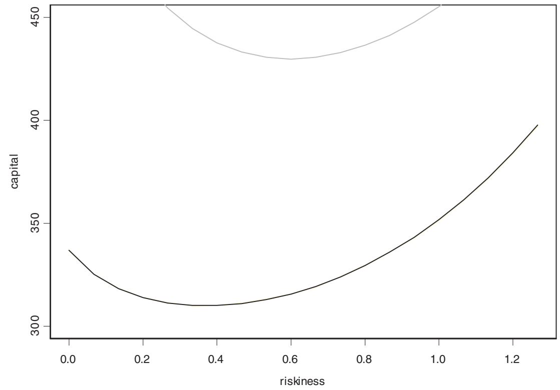

Figure 6: The 0.95-quantile of the cumulative distribution function with respect to the riskiness of an investment strategy (var = 25).

6 Conclusion

In this paper I tried to evaluate the effects of increased uncertainty in the business environment on the choice of the optimal investment strategy of conservative financial institutions such as banks, insurance companies and pension funds. As the results of two demonstrative examples indicate increasing the volatility of future losses/payments (e.g. future losses for insurance company and payments into a pension fund) of the financial intermediary will in general result in a more risky investment strategy even for a very risk averse optimization criterion. This could indicate that small companies, which have in general a higher coefficient of variation of payments or losses than bigger companies, should hold more risky assets than the bigger companies.

One should note however that the results of the examples considered are strongly affected by the choice of assumptions of the model. One such example is the duration of the investment horizon. Namely, the effect of shift towards riskier and higher yielding investment strategy is more pronounced in the case of longer investment horizon (i.e. for products and lines of businesses with longer duration) since in this case the benefits of riskier investment strategy become more pronounced. Special care should also be devoted to the assumption about the distribution of stock returns. If one assumes that the distribution of stock returns is more heavy tailed than the lognormal model predicts, the benefits of going towards risky investments will be less attractive (especially under very risk averse optimization criterion) since a larger number of cases with the above average realizations of the vector of payments/losses will be "worsened" by poor investment outcomes.

On the other hand it seems that the choice of the distribution function for losses/payments does not strongly effect the general conclusion of the simulations.

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